## **Chaos Theory and The Game of Chaos**

Chaos theory is a study that concerns the predictability of a system. It destroys the intuition that everything would be predictable if the initial condition is known. The characteristics of a chaotic system were proposed by Robert L. Devaney.

1. it must be sensitive to initial conditions,

2. it must be topologically transitive,

3. it must have dense periodic orbits ("Chaos Theory," n.d., p. 1).

A more concise definition of chaotic systems is a system that can only be described as a system of interrelated differential equations ("Chaos1," n.d.). There are some characteristics of a chaotic system. A chaotic system must be sensitive to initial conditions, which means that the small change of the initial condition of the system would result in a large change to the final condition we observe. Chaotic systems have unstable periodic properties, which means that the conditions of the system will roughly repeat itself, but it does not meet the exact location and repeat with an exact period. This phenomenon results in a dense periodic orbit. The attractor in the center of the trajectory of a chaotic system is also unstable, resulting in a strange attractor ("Attractor," n.d., p. 1). A strange attractor has a fractal structure in which infinitely many details can be seen when zooming in ("Fractal," n.d., p. 1).

Famous examples of chaotic systems include double pendulums, the Lorenz Attractor, and the n-body problem. One special case of the problem involving chaotic systems is the three-body problem (a special case of an n-body problem). Given the complexity of the force equations involved in a three-body system, it is unrealistic to solve the location of each planet after some time t given all initial conditions because there is no closed-form solution for this problem ("Three-body Problem," n.d., p. 1). Therefore, a computational simulation of a three-body system would be helpful for understanding the physics in the n-body system as well as the chaos theory.

This project, an interactive three-body simulation game called The Game of Chaos, can be used as a teaching tool for studying the principles of chaotic systems. In the logistic map generated by my program, students can discover some fundamental properties of chaotic systems.

The Game of Chaos is a cooperative game that helps students to understand chaotic systems. The goal of the game is to create a stabler n-body system as possible. Each student should receive a mass and select the location and the velocity of a planet. The score is calculated as the negative standard deviation of the positions from different initial conditions of the last planet.

By playing around with initial configurations, students could discover that adding a massive star in the center of the other stars make the system more stable. This in line with the theory that a three-body system can be approximated as a two-body system if some forces created by one or more planet can be ignored ("Three-body Problem," n.d., p. 1). Because of the program generates different mass for different students, cooperation becomes important.

After plotting the distance of the planet to the origin as a function of initial velocity, students can discover that the system has repetitions, dense periodic orbits, fractal structure, and sensitivity to initial conditions. Firstly, the distance between the final position of the planet and the origin oscillates repetitively. Secondly, as the graph zooms in, students can see more details in the range selected, but they may still see randomly plotted dense dots under or between the periodic trends, creating densely plotted regions. Thirdly, students may discover that a relatively small change to its initial velocity can result in a sudden jump in the final position to the origin. Students themselves can analyze the characteristics above in the graph generated by the program to understand the systems they created.

After generating the 3D graph showing the trajectory of the planet after some time, students can discover strange attractors. Unlike fixed-point attractors and limit cycles, the attractor in the 3D simulation move as the simulation runs with an unpredictable pattern.

If students discover that the graph generated can be model as a linear or quadratic formula. Great! The system is no longer considered chaotic and they win the game.

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